SHORT-TIME HEATING OF A BODY BY A RADIANT FLUX

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An approximate computation is proposed for heating a massive optically dense body in a furnace with predominant thermal radiation mechanism.

Heating of a massive body with a flat surface is computed in this paper. The exact values of the dimensionless body surface temperature for b = 1 are represented in the form of nomograms [1] and depend on two arguments Bo and Fo. An approximate solution is proposed in [2] in which the concept of variable depth of the layer being heated with a parabolic temperature change is proposed. In contrast to the exact solution, this solution depends on one argument $z = 1.5\mathrm{Fo}/\mathrm{Bo}^2$. The characteristic dimension Δ_* enters the numbers Bo and Fo but it vanishes in the argument z. An ultimately simple graphical solution is given a foundation there. It is shown in [3] that both approximate solutions are conserved when taking account of different emissivities and absorptivities of the furnace space. The new number b is here introduced in the dimensionless parameters. Unfortunately, the analytic solution is obtained sufficiently awkward, while the graphical solution is suitable just for computations by hand. An analytical solution is proposed below that uses the concept of the thickness of the heated layer with a parabolic temperature change from within, as in [2, 3]. However, the thickness is now included in the computation directly, whereupon it changes completely. In particular, the argument is expressed as follows $D = \sqrt{z/1.5} = \sqrt{\mathrm{Fo}}/\mathrm{Bo}_e$. The quantity z has the meaning of dimensionless time, similarly to Fo, under given boundary conditions. Correspondingly, D is proportional to the square root of the heating time. If the change in boundary conditions with the temperature of the medium is taken into account, then $D \sim T_e^3\sqrt{\tau_e}$. Evidently the doubling of the temperature of the furnace medium is equivalent to a 64 times rise in the exposure time. At first glance, the dimensionless body surface temperature $\beta_0 = T_0/(bT_e)$ is determined for a given initial value $\beta_* = \beta_{\min}$ that is identical along the depth of the body. The analytic solutions in both approximate methods are obtained implicitly.

In the formula the thickness of the heated layer is

$$\Delta_m = c\sqrt{a_0\tau} \tag{1}$$

for boundary conditions of the first kind [4], $c_1 = 6/\sqrt{\pi} =$ 3.385. For boundary conditions of the second kind $c_2 = 1.5/\mathrm{ierfc}\ 0 = 2.6586$.

In the first case the heat flux has an infinitely large value initially, and then diminishes rapidly. In the second its value is constant. Under real conditions, the heat flux can be taken constant only in a short initial period. It then diminishes in connection with the rise in body surface temperature. Therefore, $c \approx c_2$ in a short initial period. Subsequent changes in the coefficient c are difficult to predict. It must be emphasized that a set of exact solutions with different parameters Bo_e and Fo correspond to this approximate solution for the value of D_1. In principle the method cannot assure an exact solution for an arbitrary selection of c with the exception of the limit $D \to 0$. It is only possible to reduce the error and expand the limits of utilization of the method by introducing the function $c(D; \beta_*)$ at least in a rough approximation. Difficulties arise in that the "exact" solutions are taken off from nomograms in [1] by extrapolating in the parameters Bo_e, Fo, and β_* with a substantial error.

The crux of the method is simple. As in [3], the boundary condition

$$q = \sigma(\bar{\varepsilon}T^4 - \bar{a}T_0^4) + \alpha_\text{к}(T - T_0) \tag{2}$$

is simplified by introducing the equivalent temperature T_e for which

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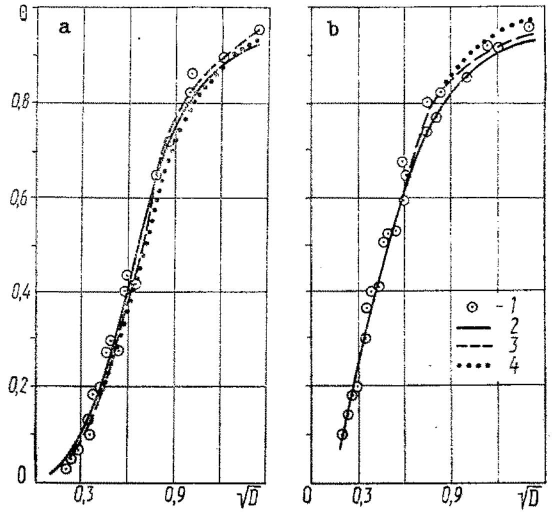

Fig. 1. Dependence of the dimensionless temperature θ on the square root of the dimensionless time $\sqrt{D}$: 1) by the nomograms in [1]; 2) by [2]; 3) by (6); 4) by (7); $\beta_* = 0$ (a); 0.75 (b).

$$q = \sigma(\bar{\varepsilon} T_e^4 - \bar{a} T_0^4). \tag{3}$$

From the equality of the right sides of (2) and (3)

$$T_e = T \sqrt[4]{1 + \alpha_\kappa [1 - (T_0/T)/(\bar{\varepsilon}\sigma T^3)}. \tag{4}$$

This exact relationship differs substantially from the approximate empirical relation

$$T_e = T \sqrt[3]{1 + 0{,}25\alpha_\kappa (\lambda_e/\Delta_*)/(\bar{\varepsilon}\sigma T^3)^2};$$

moreover, utilization of the temperature T_0 to be determined in the right side of (4) presumes an iterative solution of the problem with subsequent refinement of T_0.

For a parabolic change in the temperature in the layer Δ_m its mean value equals $\Delta T_0/3$. The heat transfer conditions do not change if the sublayer is considered isothermal with temperature T_0 and thickness $\Delta_0 = \Delta_m/3$. The average heat flux density on the surface is written phenomenologically

$$\bar{q} = \rho c_M \Delta_0 \Delta T_0/\tau$$

or, taking (1) into account

$$\bar{q} = (c/3)\sqrt{\lambda_e \rho c_M}\, \Delta T_0/\sqrt{\tau}.$$

The instantaneous heat-flux density is written in the form

$$q = \bar{q} + \tau \frac{d\bar{q}}{d\tau} = \frac{c}{6}\sqrt{\lambda_e \rho c_M}\left(\frac{dT_0}{d\sqrt{\tau}} + \frac{\Delta T_0}{\sqrt{\tau}}\right). \tag{5}$$

The equality of the right sides of (3) and (5) results in the differential equation

$$\frac{d\beta}{d\sqrt{u}} + \frac{\Delta\beta}{\sqrt{u}} - \frac{6D}{c}(1 - \beta^4) = 0, \tag{6}$$

where $0 \leqslant u \leqslant 1$, $\Delta\beta = \beta - \beta_*$, $\beta_* = \beta|_{u=0}$.

Let us use the notation $\beta_0 \equiv \beta_{max} = \beta|_{u=1}$.

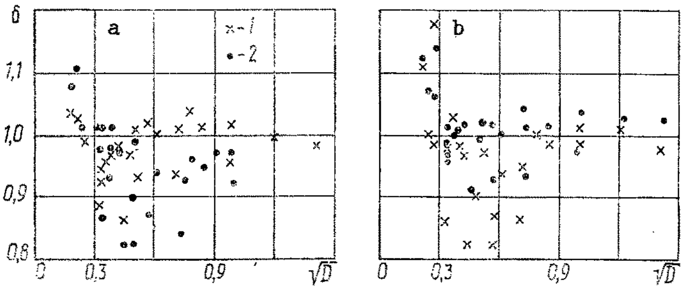

Fig. 2. Spread in the approximate solutions with respect to the "exact" values taken off the nomograms in [1]. δ: 1) from (6); 2) from (7); $\beta_* = 0$ (a); 0.75 (b).

Equation (5) is exact for $q = \bar{q} = \text{const}$ when $c = c_2$. It is used as an approximate equation for the change in heat flux density with temperture T_0 of the heating surface with average value c.

The equality $q = \bar{q}$ means $d\beta/d\sqrt{u} = \Delta\beta/\sqrt{u}$. Equation (5) simplifies into two versions. In the first $d\beta/d\sqrt{u} = 3D(1-\beta^4)/c$ and the solution takes the form

$$\frac{12D}{c} = \ln\frac{1+\beta_0}{1-\beta_0} + 2\,\text{arctg}\,\beta_0 - C,$$

where

$$C = \ln\frac{1+\beta_*}{1-\beta_*} + 2\,\text{arctg}\,\beta_*.$$

In the second version $3D/c = (\beta_0 - \beta_*)/(1-\beta^4)$, where $\beta_* \leqslant \beta \leqslant \beta_0$. In the approximate solution we set $\beta^4 = n\beta_0^4$ and we determine the coefficient n by adjustment to the exact solution. For n = 1 and $c = c_2 = 1.5\sqrt{\pi}$ $(b = 1)$ this formula is obtained directly from the solution in [4, p. 125]. Finally

$$\frac{\beta_0 - \beta_*}{1 - n\beta_0^4} = \frac{3D}{c}. \qquad (7)$$

In a first approximation we use

$$c = 2.66 + 0.1\sqrt{D}/(1+\beta_*/2), \quad n = 0.65 + 0.35\beta_*.$$

The data in [2], the numerical solution of the differential equation (6), and that obtained by the simplest formula (7) are compared in Fig. 1. The spread in the points 1 is explained by their dependence on the two arguments Bo_e and Fo in contrast to just the D in ours, and $z = 1.5D^2$ in [2]. The spread in the approximate solutions relative to the "exact" solution is shown in Fig. 2. It increases because of the fact that values with not more than two significant figures can successfully be taken off the nomograms in [1]. On the whole, (7), which is recommended for practical engineering computations, yields results no worse than the other approximate methods.

The curvature of the particle surface is taken into account by using the factor γ in the parameter $D \to \gamma D$. For a flat surface $\gamma = 1$, for cylinders and spheres with two terms in the series expansion $\gamma = 1 + \Delta_m\sqrt{\pi}/(2cd)$ and $\nu = 1 + \Delta_m\sqrt{\pi}/(cd)$ respectively. Upon substituting Δ_m from (1) $\gamma = 1 + \sqrt{\pi a_0 \tau}/(2d)$ and $\gamma = 1 + \sqrt{\pi a_0 \tau}/d$. As we see, the temperature of a convex surface increases more rapidly. But the estimates are valid only for uniform exposure of the particle surface, which is not usually assured in practice.

In high-temperature zones of rotating furnaces up to 85% of the heat is transferred to the charge through the open surface of the layer. The mean time of exposure of the surface elements of the interspersing particles is about a second. Nevertheless, the danger of a strong reduction in heat flux is expressed in connection with the rise in surface temperature.

Observations are made in [5] on the temperature of nepheline cake grains in the drum of a laboratory apparatus. The pyrometers were directed at the upper and lower edges of the layer surface. The source of irradiation was on the drum axis. The temperature radings were explicitly exaggerated. Computations we made do not have sufficient accuracy primarily because of the rough estimate for the mean time of exposure of particles in complicated motion. Nevertheless, it can be concluded that a noticeable, although not very substantial reduction in the heat flux is obtained.

NOTATION

a_0, effective thermal diffusivity coefficient, m^2/sec; $\bar{\varepsilon}$ and $\bar{a}$, reduced values of the emissivity and absorptivity of the furnace space; λ_e, effective heat-conduction coefficient, W/(m·K); c_m, specific heat of the body, J/(kg·K); ρ, body density taking account of its porosity, kg/m^3; T and T_e, actual and equivalent, from (4), temperatures of the furnace medium; T_0, body surface temperature, °K; Δ_m, depth of body heating from (1); Δ_*, characteristic dimension of the body being heated, m; τ and τ_0, running and total exposure time, sec; q, heat-flux density on the body surface, W/m^2; α_c, convective heat-transfer coefficient, $W/(m^2\cdot K)$; $\varkappa$, $W\cdot sec^{1/2}/(m^2\cdot K)$; $\sigma=5.67\cdot 10^{-8}$ $W/(m^2\cdot K^4)$. $u=\tau/\tau_0$; $b=\sqrt[4]{\bar{\varepsilon}/\bar{a}}$; $\beta=T_0/(bT_e)$; $D=\sqrt{Fo}/Bo_e=\bar{\varepsilon}\sigma T_e^3\sqrt{\tau_0}/(b\varkappa)$; $Fo=a_0\tau/\Delta_*^2$; $Bo_e=b\lambda_e/(\bar{\varepsilon}\sigma T_e^3\Delta_*)$; $\theta=(T_0-T_{0m})/(bT_e-T_{0m})$; $\beta_*\equiv\beta|_{T_{0m}}$; $\beta_0\equiv\beta|_{T_{0max}}$.

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DETERMINATION OF GENERALIZED ANGULAR COEFFICIENTS WITH CONSIDERATION OF SELECTIVITY IN ABSORPTION BY THE MEDIUM

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UDC 536.3

Generalized angular coefficients are calculated with consideration of selectivity in absorption by the medium which consists of gaseous CO_2 and H_2O.

In many technological devices which use natural fuels as an energy source radiant heat exchange is determined to a significant degree by the radiating properties of the gases CO_2 and H_2O. As many studies [1-5] have shown, these gases emit and absorb radiation with significant selectivity. However, consideration of selectivity in heat-exchange calculations for a system of bodies is an extremely difficult problem, because of lack of knowledge of the dependence of the absorption coefficient of mixtures of these gases k on frequency ν and c due to problems of a purely computational character. As a rule, technical calculations employ a selective-gray approximation, dividing the entire spectrum of thermal radiation into absorbing and nonabsorbing bands. However, such a method leads to a significant increase in the volume of calculations due to the increase in the number of zonal equations. Below we will demonstrate how selective absorption of the gaseous medium can be considered by calculating generalized angular coefficients.

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